Financial Reporting and CCR/Information Systems. Bank of Spain

11.07.2011

Precision, rounding and margin of error in reporting processes

Eurofiling CX-108

INDEX

1 Introduction

The purpose of this document is to provide some insight on concepts like precision, rounding, tolerance and margin of error that, despite of being an intrinsic part to most common mathematical operations, are sometimes confused and its effect disregarded in reporting processes.

2 Concepts

2.1 Approximated values and rounding

Although at a theoretical level most mathematical operations (and certainly all arithmetic operations) are defined accurately, in most real life tasks, the precise result of an operation is often replaced by an approximated value with a limited number of digits. This is known as rounding (the value chosen is the closest to the original value) or truncation (the value is obtained just by discarding the least significant digits). Some examples follow:

1. *Limited number of decimal digits in currencies*. The result of:

   15% x 155.10 is 23.265

   However, if this were a monetary amount in Euros, the result would be usually rounded to cents (given that currencies do not consider an undetermined number of decimals):

   15% x 155.10€ = 23.265€ » 23.27€
2. *The result of some operations cannot be expressed using a finite number of decimal digits*. For instance:

   10 / 3 = 3.333333…

   Ö2 = 1.41421356…
3. Some *information exchange protocols* impose *constraints* on the number of digits used to represent a certain amount in order to reduce the size of the file exchanged and thus, reducing the use of bandwidth.
4. Similar restrictions are sometimes found in databases. As a consequence, amounts are rounded or truncated to a certain number of decimal digits
5. Despite the fact that previous examples refer to rounding applied to decimal digits, this idea can be generalized to other digits (units, tens, hundreds, thousands). A typical example are data entries in electronic forms, which are usually defined in terms of a multiplication factor in order to make easier the task of filing the data manually. For instance, in an entry like the one that follows, the user will round the amount to thousands:

Assets (thousands of €): 5777

2.2 Tolerance and precision

In most scenarios, dealing with approximated values is more than enough, considering that the deviation of the approximated value from the theoretical one is very small and consequently, it doesn't have an impact on the practical outcome of a certain task.

However, in different scenarios, it is important to have more detailed information. As dealing with accurate values is not always feasible, at least, it is important to have information not only about the amount itself (referred to as nominal value), but also about its accuracy.

There are different ways to express the accuracy of an amount:

6. Tolerance, sometimes called margin of error, indicates the maximum difference (in absolute terms) of the nominal value and the theoretical value. The tolerance is often expressed as a percentage (relative tolerance), but can also be expressed in absolute terms:
   - 5,745 ± 5% means that the difference between the accurate value and the nominal value is, at most, 287.25 (the 5% of 5,745).
   - 7,230 ± 0.1 means that the nominal value (7,230) can differ at most 0.1 from the accurate one.

Thus, the tolerance defines an interval of possible values. The values at the edges of the interval defined are called endpoints. For instance, 5,745 ± 5% defines an interval whose endpoints are 5457.75 (5,745 - 5%) and 6032.25 (5,745 + 5%).

7. Precision: indicates the number of digits of a certain amount that are known to be equal to the accurate value. For instance:
   - 2,500 precision = 2, means that the first two digits of the nominal value (2 and 5), are known to be exact. So, there could be differences in tens, units and decimal digits.
   - 2,500 precision = 4, means that the first four digits of the nominal value are known to be exact and thus, there could be differences in decimal digits.
8. Precision is sometimes expressed in terms of a number of decimal digits. For instance:
   - 2,534.7 decimals = 1, means that one decimal digit plus all the digits at the left side of the decimal point are known to be exact.
   - This number can be negative: 25,000 decimals = -3 means that the first three digits at the left side of the decimal point are not accurate. So, only tens and units of thousands are known to be accurate.
   - 25,777.33 decimals = -3 also means that only the first two digits are know to be accurate. Some people wrongly believe that an example like this is not consistent, i.e., that digits not covered by the precision should be equal to zero. The example is correct: *the nominal value represents the better approximation to the theoretical value that can have been obtained; the precision (or decimals, or tolerance) is a measure of the accuracy of the nominal value*.

2.2.1 Operations with precision/tolerance

The tolerance of the result of an operation can be calculated by considering all possible results obtained by combining the endpoints of each operand. For instance, let's say that we want to calculate the result of the operation A + B, where A is 250,000 (± 500) and B is 57,250 (± 1):

250,000 (± 500) + 57,250 (± 1) = ?

A	B	A + B
249,500 (= 250,000 – 500)	57,249 (= 57,250 – 1)	306,749
250,500 (= 250,000 + 500)	57,251 (= 57,250 + 1)	307,751
249,500 (= 250,000 – 500)	57,251 (= 57,250 + 1)	306,751
250,500 (= 250,000 + 500)	57,249 (= 57,250 – 1)	307,749

The highest and lowest possible values are 307,751 and 306,749. These two values constitute the endpoints of the interval result of the operation, which can also be expressed as 307,250 ± 501. In fact, this result could have been obtained like this:

Nominal value of A + B = Nominal value of A + Nominal value of B

Tolerance of A + B = Tolerance of A + Tolerance of B

As a matter of fact, the nominal value of any operation can be obtained by performing the operation as it would be done with ordinary numbers. However, the tolerance follows different rules. For instance:

Nominal value of A x B = Nominal value of A x Nominal value of B
Tolerance of A x B = Tolerance of A x Nominal Value of B
+ Tolerance of B x Nominal Value of A
+ Tolerance of A x Tolerance of B

2.2.2 Relationship between precision/tolerance and rounding

Precision and rounding are two different concepts that should not be confused: the precision is a measure of the accuracy of data whereas rounding is an operation applied as a consequence of a limitation in the number of digits supported by certain tasks.

Rounding operations usually[1] produce a decrease in the precision of the data. An accurate amount that is rounded to thousands will have a tolerance of ±500. For instance: 5,257,233.55 » 5,257,000 ±500.
However, data with a certain precision does not have to be rounded. For instance:

9. 9% x 32,500 ±500 » 2,925 ± 45

2.3 Margin of error

The precision plays an important role when the consistency of data is to be verified using validation rules. Disregarding the precision of the data involved might lead to false consistency warnings, as the following example shows: let's suppose that some data is filed by an electronic form; A and B are monetary amounts, and C is a ratio known to be higher than the quotient of A and B (C > A/B):

	Original data	Reported data
A	45,250	45,250
B	25.49	25
C	1,780	1,780
A/B	1,775.21	1,810

The original amounts are correct: the quotient of A and B is lower than C. However, the check will fail with reported data. The source of the problem is that the precision of the data has not been considered. If the tolerance of the quotient is obtained, we will see that the condition is also satisfied for reported: the lower endpoint of A/B (»1,773) is lower than the reported amount C.

	Original data	Reported data
A	45,250	45,250 ± 0.50
B	25.49	25 ± 0.50
C	1,780	1,780 ± 0.50
A/B	1,775.21	» 1,810 ± 37

[1] This loss of precision does not strictly occur always. For instance, 275,000 ± 0.5 rounded to thousands retains its original precision.

In other words, the comparison applied in consistency this check (C > A/B) takes into account a margin of error that can be obtained from the tolerance of the operands:

1,780 ± 0.50 > 1,810 ± 37 => 1,780 > 1,810 ± 37.50 => 1,780 > 1,810 - ***37.50***[2]

In this example, 37.50 constitute the margin of error applied.

3 Precision in supervisory reporting processes

There are two different situations where the precision of the information exchanged in supervisory reporting processes must be considered: the first one it is related to the accuracy of the data itself and the second one is related to the validation process:

1. Banking supervisory authorities need to asses the situation of credit institutions, the banking sector or any other subject of their competence. If the *assets* of a certain institution were filed as 3,500,000±2,000,000€, it would be very difficult to express any opinion about its financial reliability. It stands to reason to require a minimum precision of the data filed.

2. Most supervisory authorities perform a validation process on the data received. Its purpose is to detect potential errors caused by misunderstandings in the meaning of the concepts required by the supervisor, by a wrong identification of the data in their databases, errors in the calculation of derived data or typos in those cases where data is filed manually. Given that the data required often presents many redundancies, it is possible to define a set of mathematical checks on the data that will help to detect this kind of problems.

In order to warrant the reliability of the analysis of the data gathered, supervisory authorities should require a **minimum precision** of data: the more precise the data, the more reliable. However, a very high precision requirement will make things difficult for supervised institutions. Though there are mathematical approaches (numeric calculus) and technical solutions (infinite precision libraries) that make possible dealing with very accurate data, these solutions might be expensive or even unfeasible, provided that data stored in the credit institutions' databases might have a certain tolerance.

This minimum precision does not necessarily have to be the same for all supervised institutions: a precision of ± 500,000€ (1 million of €) in an amount of 1,110,529 millions of euros (Santander Group's assets in 2009) implies a relative tolerance of a ±0.000045%; however, a precision of ± 0.5 (1 €) in an amount of 2,500€ (the order of assets of some currency exchange companies in Spain) implies a relative tolerance of ±0.02% (a precision around 500 times worse than the previous example).

Regarding the validation process, the precision of data must be considered as it was described in chapter 2.3: margin of errors should be used in all those checks that compare data. If the margin of error used in a certain check is to high, wrong data will pass undetected; in the margin of error is too small, correct data will produce false warnings making complex its diagnostic and reducing the transparency of the process. The margin of error is a topic that has been covered thoroughly by

[2] In this operation, the positve value of the margin of error is the more restrictive, so it should be discarded.

mathematical methods like interval arithmetic and now is available in our technology and standards (IEEE 2008). Whenever possible, such kind of methods should be applied to optimize the effectiveness of the validation process.

3.1 Answers to some common questions

3.1.1 Should data be rounded before being sent to a supervisor

Unless there is an explicit requirement from domain experts to round or truncate some of the concepts required, data should not be modified. There are no technical benefits in rounding the data in modern data formats like XBRL; quite the contrary, rounding implies a loss of precision, a manipulation of the data and a bigger effort for the filer. Ideally, data sent to a supervisor should not suffer any modification, manual or automatic in order to improve the transparency of the reporting process and its traceability. Data should be sent with all the precision available in the source systems.

3.1.2 Should a fixed precision be required?

If a fixed precision is required, unless the precision in credit institutions' systems happened to coincide with the precision required, we would face different situations depending on the approach taken by the filer:

- The precision in the filer's systems is higher and the data is rounded: loss of precision, manipulation of the data and less traceability (see previous paragraph on rounding).
- The precision in the filer's systems is higher, the nominal value is not modified, but the reported precision is changed as required. In this case, the truth is somehow distorted: though the data has a certain precision, the reported precision is a different; data is manipulated as well (only its precision, not its value), and so, there is a bigger effort on the filer's side and a certain loss of traceability.
- The precision in the filer's systems is *lower*, the nominal value is not modified but the reported precision is changed as required. Like the previous case, the truth is distorted, but in a more dangerous way, as the real precision is lower than the reported one.

So, there are no apparent benefits in forcing a fixed precision in reporting processes. In fact, a fixed precision is something unnatural to processes where data is aggregated in different stages. For instance, the precision of the consolidated data of a group of companies will be lower (in absolute terms) than the precision of the data of its individual companies (the tolerance of each individual company adds to the tolerance of the group); and the precision of the banking sector will be lower than the precision of groups.

3.1.3 So, what is the precision of the data that should be required on a regular basis?

The precision of data should represent just that: the precision of the data available in the credit institution. If required by business areas, supervisors should impose a *minimum precision* to warrant a minimum accuracy of the data analyzed, and possibly to keep compatibility if existing systems presume a certain minimum accuracy of the data to perform some additional checks. Any other constraint will increase the reporting burden for the credit institution, produce a reduction of the accuracy of the data, favour its manipulation and distortion, and reduce the traceability and transparency of the process.

3.1.4 *Is it convenient to establish a common minimum precision in the different steps of the reporting chain?*

No, it is not. A minimum precision (in absolute terms) that might fit perfectly the requirements for data corresponding to big European credit institutions, might not be enough for smaller institutions or individual information at national level (see example in the introduction of chapter 3). On the other hand, the precision requirements for small institutions cannot be imposed to big ones, as the tolerance of the amounts of the big ones is the sum of the tolerance of many individual amounts.

However, there must be coherence in the minimum precision required throughout the reporting chain: the precision requirements of a step cannot be softer than the requirements of a subsequent step. For instance, if the required minimum precision for amounts of credit institutions in the exchange of data between EBA and NSAs is ± 2.500 €, the required minimum precision for those credit institutions at national level cannot be lower (lower precision / higher tolerance) than that; it must be equal or higher (for instance, a precision of ± 50 €).

3.1.5 *Wouldn't it be easier if derived data were calculated after being rounded?*

What if instead of preparing the validation process at the supervisor side to deal with the burden of precision and margins of error, we just require the filers to round data to a certain number of digits and ask them to obtain derived data from rounded data? Apparently, following this approach, there is no need to deal with precision problems and we could just apply simpler validation checks that do not care about margins of error. In the following example, concept C is the addition of A and B. If C is obtained from the original data and rounded, unless the margin of error is considered, an error will be raised. However, in the case of C obtained from rounded data, this margin is not necessary.

		Rounded data	
Concepts	**Original data**	**C obtained from original data and then rounded**	**C obtained from rounded data**
A	45,236.25	45,236.00	45,236.00
B	75,252.42	75,252.00	75,252.00
C (A+B)	120,488.67	120,489.00	120,488.00
Expected C	-	120,488.00	120,488.00

At a first glance, this approach might seem straightforward; however, it hides some important issues:

- Data calculated after rounding is less accurate than data calculated before rounding.
- It increases the reporting burden in credit institutions: they are expected to build a calculation engine for data they might already have available in their systems, with better accuracy.
- The calculation of derived data from the validation rules defined by the supervisor is not a straightforward problem. The following example shows how two different validations rules applied to the same concept (A = A1 + A2 + A3 + A4 + A5 and A = B x C), could lead to two inconsistent results:

		Rounded data	
Concepts	Original data	A obtained from original data and then rounded	A obtained from rounded data
A (A1 + ... + A5)	61.18	61.00	59
A1	4.4	4.00	4.00
A2	15.4	15.00	15.00
A3	23.45	23.00	23.00
A4	5.46	5.00	5.00
A5	12.47	12.00	12.00
A = B x C	61.18	61.00	66.00
B	5.5	6.00	6.00
C	11.124	11.00	11.00

- Moreover, this approach is limited to equality validations (A = B + C, A = U x V). Other type of comparison validations cannot be used to derive data (e.g.: A < B + C).

3.1.6 If precision is not fixed, how can a common set of validations work on data with variable precision?

The error margin used in a check must be obtained considering the precision of input data. As it was mentioned before, the calculus of the margin of error is covered by mathematical methods like interval arithmetic and is available in current technology and standards. The idea is depicted in the following example:

Let's say that we want to check that a figure A is equal to the addition of B, C and D:

$$A = B + C + D$$

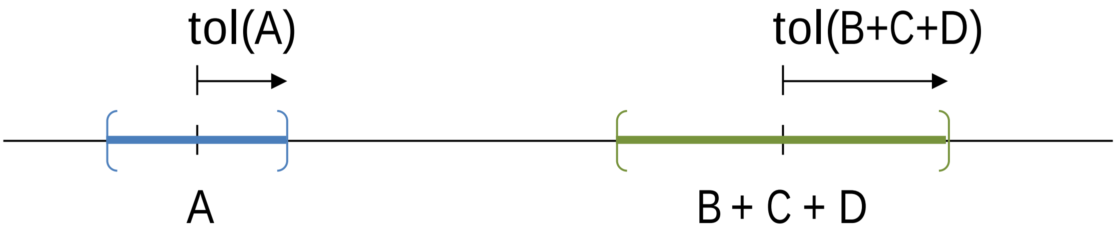
We must consider that every amount reported has a tolerance[3]. So what we do really have is:

$$A \pm tol(A) = (B \pm tol(B)) + (C \pm tol(C)) + (D \pm tol(D))$$

That can be simplified like this:

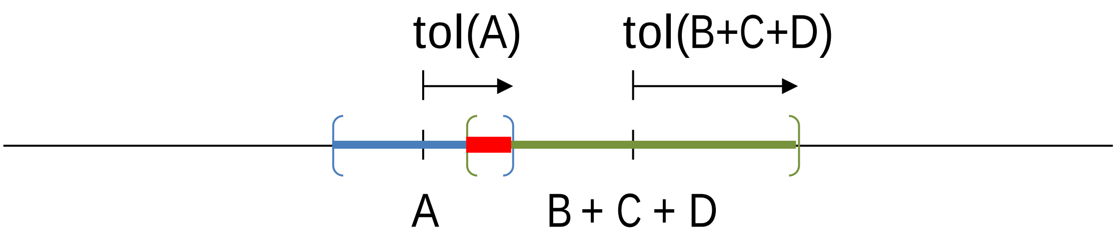
$$A \pm tol(A) = B + C + D \pm (tol(B) + tol(C) + tol(D))$$

So, each part of the comparison expression has an interval of possible values with a central value (A on the left side of the equal and B + C + D on the right side of the equal). These intervals must be considered in order to deem if the values reported are within the range of accepted values. This can be represented graphically like this:



If the intervals do not overlap, then we can consider that there is a mistake in the values reported. If the intervals do overlap, then, data is valid according to this check:

[3] In the examples tol(A) means the tolerance of the amount A.



This graphical check can be expressed as follows:

$$abs(A - (B + C + D)) \leq tol(A) + tol(B) + tol(C) + tol(D)$$

If the tolerance of the amounts reported is of thousands of Euros (± 500€), then we will have:

$$abs(A - (B + C + D)) \leq 2.000\text{ €} \quad (4 \times 500\text{ €})$$

If the tolerance of the amounts reported is units of Euros (± 0.5 €):

$$abs(A - (B + C + D)) \leq 2\text{ €} \quad (4 \times 0.5\text{ €})$$

With this approach, the margin of error applied is obtained from the precision of the input data and the operations performed. The margin or error will be shorter for data reported with a higher precision and longer for the reported with a lower precision. The same check can be applied at any step of the reporting chain, no matter the precision of the data.

3.2 Conclusions

In order to improve the transparency of the reporting process, we should avoid putting constraints that would force institutions to manipulate the data in their systems and that would lead to a loss of accuracy of the information sent. In those reporting processes based on format like XBRL[4] that support the use of precision, it should reflect the accuracy of the data managed by the institution and not a predefined value established arbitrarily. A minimum precision is advised to be required in those reporting processes that need a minimum accuracy of the data in order to carry out successfully their tasks. Validation processes should apply standard methodical approaches to obtain the margins of error allowed depending on the kind of operations performed and the precision of input data. This approach enables the detection of the maximum number of errors in the data reported but limiting the amount of false warnings.

Ultimately, the decision of choosing a minimum accuracy of data and the approach to deal with margins of error is something to be determined by functional areas (with the proper support of technical ones), given the direct impact that these decisions have on the reporting process.

Regarding validations, there are technical solutions that consider the precision of the data reported in order to apply a certain margin of error. This enables the possibility of using a common set of validations to check data reported with different degrees of precision.

[4] The attribute decimals should be used for this purpose